Journal of Statistical Physics, Vol. 77, Nos. 1/2, 1994

Note on the Two-Slit Experiment

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Received July 30, 1993; final September 8, 1993

According to the general philosophy of quantum mechanics, a particle whose passage through one of the slits of a two-slit apparatus has been detected does not produce interference. In a previous article this was demonstrated explicitly by solving the Schrödinger equation for a specific model of the detector, but only the first order in the interaction with the detector was considered. In the present note it is shown for the same model that for stronger interactions the interference disappears altogether. When the detector has reached 100% efficiency those particles that have not been detected do not produce interference either, because they are sure to have passed through the other slit.

KEY WORDS: Quantum measurement.

1. INTRODUCTION

When a monoenergetic beam of electrons passes through two parallel slits an interference pattern is produced. According to the celebrated discussion of Bohr,[1] this pattern disappears when the passage of the electron through the upper slit is detected. The wave function then no longer consists of two coherent waves emerging from the slits, but collapses into one wave emerging from the upper slit. In a previous article[2] a physical model of a measuring process has been constructed in which this collapse could be demonstrated explicitly. Thus the collapse is *not* an added postulate, but a consequence of the Schrödinger equation for the combined system of electron plus detector, including their interaction.[3]

I used the word "detector" for a measuring apparatus whose outcome is "yes" or "no." Whoever wants to construct an explicit model of a measuring apparatus must bear in mind that it has to be *macroscopic* in order to make a permanent registration possible.[4] Moreover, it has to be

For Oliver Penrose, as a token of esteem.

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822/77/1-2-24

prepared in a *metastable* state in order to magnify the microscopic event into a macroscopic signal.[5] In our model the detector is an atom together with the electromagnetic field. The atom is prepared in its metastable $2S$ state and initially the field is empty. The passage of the electron is detected because its Coulomb field perturbs the $2S$ state and thereby triggers a transition to the $1S$ ground state while emitting a photon. Our detector is macroscopic because of the large number of modes of the field; this has the consequence that the transition is irreversible, since the photon cannot return to reexcite the atom.

The equations for this model were formulated in ref. 1, but they were not solved, as that was not needed for the discussion. The discussion, however, involved only the first order in the interaction. In this note we obtain the solution to all orders, at the expense of some further simplifications of the model.

2. THE MODEL

The state of the combined system is a Hilbert vector Ψ in the product space of electron and apparatus, the latter being itself the product of the two-dimensional Hilbert space of the atom and the Hilbert space of the radiation field. For our purpose we de not need all apparatus states, but only:

- The state $|2S;0\rangle$: excited atom, no photons.
- The states $|1S;\mathbf{k}\rangle$: atom deexcited, one photon $\mathbf{k}$.

They form an orthogonal, though incomplete, basis in the Hilbert space of the detector. The total state of the combined system is a linear combination of these states with coefficients that are functions of the coordinate $\mathbf{r}$ of the electron:

$$\Psi = \varphi(\mathbf{r})\,|2S;0\rangle + \sum_{\mathbf{k}} \psi_{\mathbf{k}}(\mathbf{r})\,|1S;\mathbf{k}\rangle$$

The norm in the combined Hilbert space is

$$\langle\Psi|\Psi\rangle = \int |\varphi(\mathbf{r})|^2\,d\mathbf{r} + \sum_{\mathbf{k}} \int |\psi_{\mathbf{k}}(\mathbf{r})|^2\,d\mathbf{r}$$

The two terms are, respectively, the probability that the atom is in the excited state $2S$ and the probability that it is in the ground state having emitted a photon.

The Schrödinger equation for a stationary state (with energy E) can be decomposed into a set of equations for the mutually orthogonal components:

$$E\varphi(r) = (\Omega - \tfrac{1}{2}\nabla^2)\,\varphi(\mathbf{r}) - iu(\mathbf{r}) \sum_{\mathbf{k}} v_{\mathbf{k}} \psi_{\mathbf{k}}(\mathbf{r})$$

$$E\psi_{\mathbf{k}}(r) = (k - \tfrac{1}{2}\nabla^2)\,\psi_{\mathbf{k}}(r) + iu(\mathbf{r})\, v_{\mathbf{k}}\, \varphi(\mathbf{r})$$

Ω is the excitation energy of the atom, k is the energy of the photon, and $-\frac{1}{2}\nabla^2$ is the kinetic energy of the electron. The function $u(\mathbf{r})$ is the dipole moment of the $2S$ state induced by the presence of the electron at the position $\mathbf{r}$, while $v_{\mathbf{k}}$ are numerical constants. The functions φ, $\psi_{\mathbf{k}}$ are the solutions of these equations, subject to the boundary conditions imposed by the screen with slits, and to the condition that only φ contains an incoming wave. Then the outgoing component of φ describes the electron when it passes without being detected; this component exhibits interference between both slits. The outgoing components of the $\psi_{\mathbf{k}}$ describe the electron in case it has been detected. They are waves spreading from the vicinity $u(\mathbf{r})$ of the atom in the upper slit, and no interference results from them. The change from φ into $\psi_{\mathbf{k}}$ is the so-called collapse of the wave function.

Four our present purpose the following simplifications[2] are made. We consider a single coordinate x, being the direction of propagation. The two components of φ going through the upper and lower slits are denoted by φ_1 and φ_2, respectively. Furthermore, $u(\mathbf{r})$ is taken to be a delta function in the upper slit. The corresponding simplified Schrödinger equation is

$$E\varphi_1(x) = \left(\Omega - \frac{1}{2}\frac{d^2}{dx^2}\right)\varphi_1(x) - i\delta(x) \sum_{\mathbf{k}} v_{\mathbf{k}} \psi_{\mathbf{k}}(0)$$

$$E\varphi_2(x) = \left(\Omega - \frac{1}{2}\frac{d^2}{dx^2}\right)\varphi_2(x)$$

$$E\psi_{\mathbf{k}}(x) = \left(k - \frac{1}{2}\frac{d^2}{dx^2}\right)\psi_{\mathbf{k}}(x) + i\delta(x)\, v_{\mathbf{k}}\, \varphi_1(0)$$

3. SOLUTION OF THE EQUATIONS

The *second* line gives trivially

$$\varphi_2(x) = e^{ip_0 x}, \qquad \text{with} \quad p_0 = (2E - 2\Omega)^{1/2}$$

[2] An elaborate calculation not involving these simplifications was recently given by A. von Peij (unpublished master thesis, Eindhoven).

The normalization is arbitrary. The *first* line gives

$$\begin{aligned}\varphi_1(x) &= e^{ip_0x} + Ae^{-ip_0x} \qquad (x<0)\\ &= Be^{ip_0x} \qquad\qquad (x>0)\end{aligned}$$

The matching conditions at $x=0$ lead to

$$B = 1 + A = 1 - \frac{1}{p_0}\sum_{\mathbf{k}} v_{\mathbf{k}}\psi_{\mathbf{k}}(0)$$

The *third* equation gives

$$\psi_{\mathbf{k}} = C_{\mathbf{k}}e^{ip|x|} \qquad \text{with} \quad p = (2E - 2k)^{1/2}$$

$$C_{\mathbf{k}} = \frac{v_{\mathbf{k}}}{p}\varphi_1(0) = \frac{v_{\mathbf{k}}}{p}B$$

Combination with the above equation for B yields

$$B = \left(1 + \frac{1}{p_0}\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}^2}{p}\right)^{-1}$$

This completes the solution.

The number of electrons per unit time in the incoming component of φ_1 is p_0, and the number of transmitted electrons is p_0B^2. In addition there are the electrons transmitted after having been detected:

$$\sum_{\mathbf{k}} p\,|C_{\mathbf{k}}|^2 = \left(\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}^2}{p}\right)B^2$$

Finally φ_2 corresponds to p_0 incoming electrons—which are all transmitted. Hence out of the incoming $2p_0$ electrons there are transmitted

$$p_0B^2 + \left(\sum_{\mathbf{k}}\frac{v_{\mathbf{k}}^2}{p}\right)B^2 + p_0 = p_0(1+B)$$

The other $p_0(1-B)$ electrons are scattered in the backward direction through the interaction with the detector.

The wave functions $\psi_{\mathbf{k}}$ do not interfere with φ because they belong to orthogonal components of the total Hilbert vector Ψ. The two functions φ_1 and φ_2, however, are coherent because they belong to the same component of Ψ; they were merely our simplified representation of the actual wave φ that passes through the double slit. Hence they are two coherent sources with amplitudes B and 1, respectively, at a mutual distance $2a$, being the distance between the slits. They produce interference lines, whose intensity in a direction with angle ϑ with the axis is

$$B^2 + 1 + 2B\cos(p_0 a\vartheta)$$

Notice that the average of this intensity is the sum $B^2 + 1$ of the two separate intensities. This is our justification for counting the number of transmitted electrons by adding up those passing through each of the slits as if there were no interference.

4. LIMITING CASES

So far no restriction has been imposed on the magnitude of the coupling constants $v_{\mathbf{k}}$. Earlier[2] they were tacitly assumed to be small. That made it possible to study the $\psi_{\mathbf{k}}$, which are of the first order in the $v_{\mathbf{k}}$, while regarding φ as unaffected, since B deviates from unity only in second order. Hence in that case the detector is so inefficient that it does not noticeably diminish the number of electrons that pass undetected. As a consequence, the intensity of the interference with φ_2 is in that approximation

$$1 + 1 + 2\cos(p_0 a\vartheta) = 4\sin^2(\tfrac{1}{2} p_0 a\vartheta)$$

The visibility, defined in ref. 6 by $(I_{\max} - I_{\min})/(I_{\max} + I_{\min})$, equals 1.

Now consider an interaction of arbitrary strength. The relevant parameter is

$$\sigma = \frac{1}{p_0} \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}^2}{p} = \frac{1}{(2E - 2\Omega)^{1/2}} \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}^2}{(2E - 2k)^{1/2}}$$

If σ is not small compared to unity, one has $B < 1$, so that an appreciable fraction of the electrons through the upper slit are detected. The remaining ones interfere with φ_2; the intensity of the interference lines oscillates between $(1 + B)^2$ and $(1 - B)^2$. The visibility equals $2B/(1 + B^2) < 1$.

In the extreme case $\sigma \gg 1$ one has $B \approx 0$; almost every electron in the upper slit is detected. The few that remain to interfere with φ_2 produce only a small ripple on the background provided by the lower slit. This situation may be described by stating that *with such an efficient detector any electron that is not detected is known to have passed through the lower slit, just as surely as if there were a detector in that lower slit as well, and therefore no interference is possible.* It was this statement, due to L. Kleinman, that prompted the work in this note.

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